A Computer Program for Calculating Internal Rate of Return and Present Net Worth Profiles[1/]

by

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Program IRR is a FORTRAN IV program that calculates the internal rate of return (IRR) for a specified series of net returns. The program also calculates present net worths (NPW) for a number of interest rates at specified intervals. It can be executed interactively over a computer terminal.

A Newton-Raphson iteration technique is used to find the internal rate of return. Given an initial interest rate i_n and the PNW-function related to i, f(i), the next interest rate i_{n+1} in the iteration is found from $i_{n+1} = i_n - \frac{f(i_n)}{f'(i_n)}$ where $f'(i_n)$ is the derivative of the function at i_n approximated in the program by the slope of a secant through the function f(i) at two points or

$$f'(i_n) = \frac{f(i_{n+1}) - f(i_n)}{i_{n+1} - i_n}$$

This process is best illustrated by examining Figure 1 where the iteration was started with the two interest rates 2 and 10 percent. The slope of the secant through the two values of f(2) = 60 and f(10) = -15, representing the net present worths at the two assumed interest rates, is given by

$$f'(10) = \frac{f(10) - f(2)}{i_2 - i_1} = \frac{-15 - 60}{10 - 2} = \frac{-75}{8} = -9.38$$

The straight line with this slope will intersect the interest rate axis at the new trial interest rate:

$$i_3 = i_2 - \frac{f(i_2)}{f'(i_2)} = 10 - \frac{(-15)}{-9.38} = 10 - 1.6 = 8.4$$

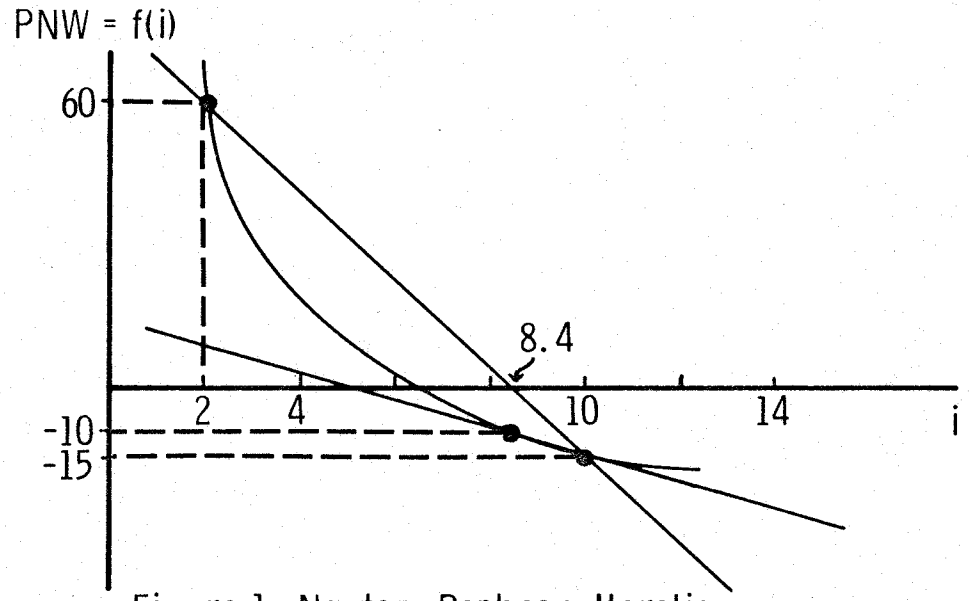


Figure 1. Newton-Raphson Iteration

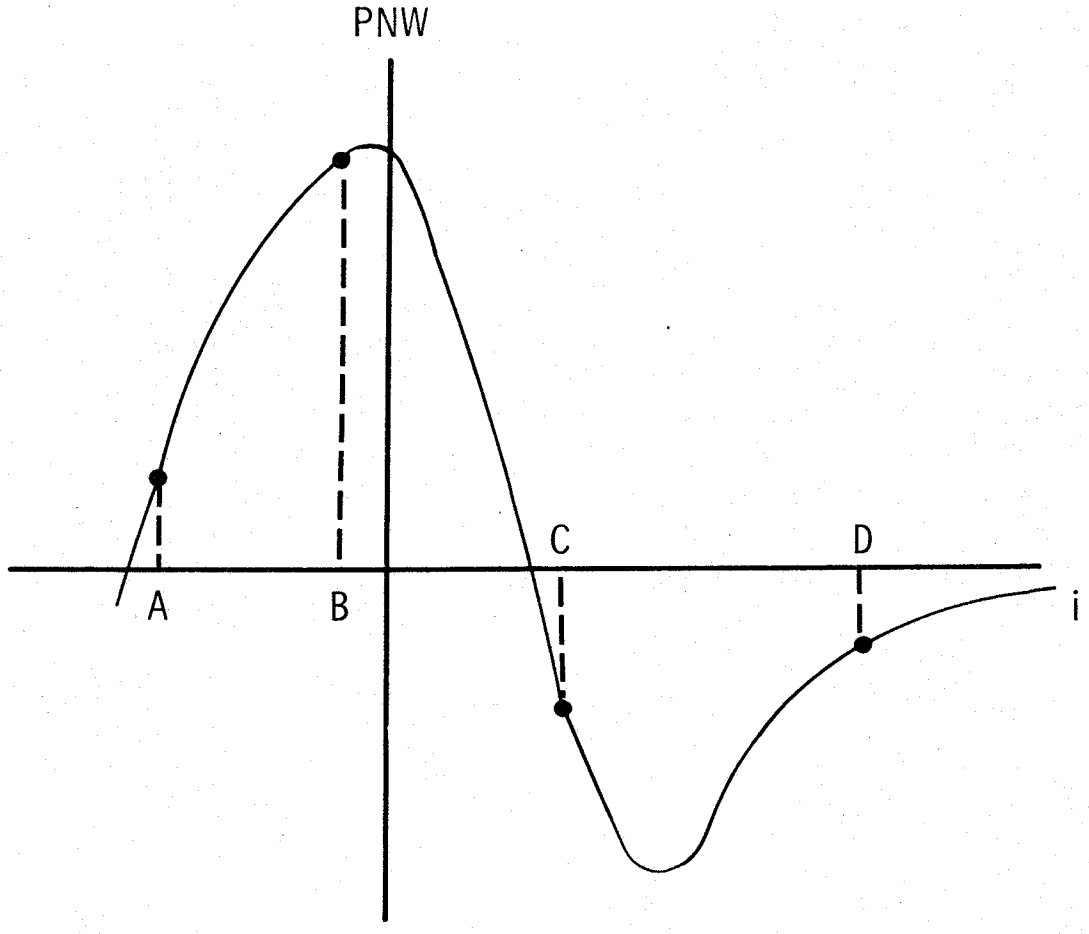


Figure 2. Possible Problems in Finding Roots of Function

and f(8.4) = -10. The next interest rate in the iteration would be 5.07. This process is continued until the functional value of present net worth is sufficiently close to zero.

Running the Program

Assuming that the compiled version of the program (IRR) has already been stored on a disk file, the execution of the program is quite simple. Shown below are control commands necessary for program execution. All user supplied inputs are underlined.

```
 78/03/02. 15.42.11.  TERMINAL - 206
ALL USERS-TYPE*WRITEUP,SYSNOTE* (78/02/23).
USER NUMBER: [redacted]
TERMINAL:    206, TTYD
PF STORAGE EXCEEDS LIMIT.
RECOVER /SYSTEM: GET,IRR
READY.
X,IRR

  NUMBER OF NET CASHFLOWS  (UP TO 4)
? 1

  NAME OF CASHFLOW  1 (UP TO 8 CHARACTERS)
? INVEST 1
```

Little has to be said about program inputs as they become self-explanatory at the time of program execution. The program when accessed via the previously described control commands will ask for relevant inputs. Examination of the attached sample run should give sufficient insight into the inputting procedure. All inputs except the names of the cashflows are free-formated, i.e., can be entered without regard to size and number of digits after a decimal point or field on the input line or card. Individual numbers must be separated by one or more blanks or a comma.

Program output consists of

(1) the internal rate of return for each specified cashflow; sometimes the iteration procedure will not be able to find the internal rate of return because of the peculiar nature of a cashflow.

(2) a cashflow profile showing present net values for specified interest rates

(3) a graphical output of the cashflow profile. This output can only be requested if a cashflow profile was produced.

Cashflow profiles such as shown in Figure 2 can present a problem in finding an internal rate of return. Note that there are two solutions at which PNW = 0 and that PNW approaches the interest rate axis asymptotically on one side. If the two initial interest rates selected for the iteration are A and B, the procedure will find one of the roots of the function. With B and C as starting values, the second root would have been found. With B and D as starting values, however, the iteration

process would solve for larger and larger new interest rates often leading to numbers that exceed the maximum word length of the computer. If an internal rate of return was not found by the program, new initial starting values for the interest rates can be used to find a solution (see sample run).

A number of program variables are initialized at values that one might want to change. OFF is a variable the value of which will determine when the iteration is to be terminated. Presently set at a value of 0.000 the iteration will stop as soon as the IRR is determined to the nearest 1/1000 of one percent as long as NPW is less than 0.1 dollar. For the latter, the parameter OFP is set equal to 0.1. The process will also be stopped if the slope (parameter SLP) becomes smaller than 50, i.e., present net worth does not change more than $0.05 per 1 percent change in the interest rate, possibly indicating an asymptotic problem.

A program listing and example output are shown below.

```
      PROGRAM IRR (INPUT,OUTPUT,TAPE1=INPUT,TAPE6=OUTPUT)
      DIMENSION RETURN(100,4), IYEAR(100,4), PNW(4), PN1(4), NAME(4), NB
     1R(4), RIT(4), RIRR(4), IY(4), KNT(4), RATE(4), PNW2(100,4), NX(3),
     2 NY(3), IMAGE(1200), IRAY(5), ICHAR(4), XAXIS(7), YAXIS(7), RATI2(
     3100)
      DATA NX/4,18,1/,NY/4,7,1/
      DATA IRAY/10H       DIS,10HCOUNTED NE,10HT REVENUES/
      DATA ICHAR/1HA,1HB,1HC,1HD/
      IPT=1
      NPT=6
      SLP=50.
      OFF=0.1
      OFF=0.0001
      WRITE (NPT,140)
      READ (IPT,) NA
      DO 90 J=1,NA
         WRITE (NPT,150) J
         READ (IPT,160) NAME(J)
         WRITE (NPT,170) NAME(J)
         READ (IPT,) IY(J)
         WRITE (NPT,180)
         READ (IPT,) (IYEAR(I,J),RETURN(I,J),I=1,IY(J))
         WRITE (NPT,190)
         READ (IPT,) RATE(J),RIRR(J)
         I1=0
         I2=10
         WRITE (NPT,200) NAME(J)
20       I1=I1+1
         IF (I1.GT.IY(J)) GO TO 30
         IF (IY(J)-5.LT.I1) I2=IY(J)
         WRITE (NPT,210) (IYEAR(I,J),I=I1,I2)
         WRITE (NPT,220) (RETURN(I,J),I=I1,I2)
         I1=I2
         I2=I2+10
         GO TO 20
30       PNW(J)=0.0
         RIT(J)=RATE(J)/100.
         DO 40 I=1,IY(J)
            K=IYEAR(I,J)
            RTE=(1.+RIT(J))**K
40          PNW(J)=PNW(J)+RETURN(I,J)/RTE
         PN1(J)=PNW(J)
C
C     NEWTON-RAPHSON ITERATION
C
         KNT(J)=0
         RIRR(J)=RIRR(J)/100.
50       RIR=1.+RIRR(J)
         PNW(J)=0.0
         KNT(J)=KNT(J)+1
         DO 60 I=1,IY(J)
            K=IYEAR(I,J)
            RTE=RIR**K
60          PNW(J)=PNW(J)+RETURN(I,J)/RTE
         DIFF=RIT(J)-RIRR(J)
         IF (ABS(DIFF).LT.OFF) GO TO 80
         SLOPE=(PN1(J)-PNW(J))/DIFF
         IF (ABS(SLOPE).GT.SLP) GO TO 70
         WRITE (NPT,340) NAME(J)
         WRITE (NPT,350)
         READ (IPT,230) REPLY
         IF (REPLY.EQ.'NO') GO TO 90
         WRITE (NPT,190)
         READ (IPT,) RATE(J),RIRR(J)
         GO TO 30
70       IF (ABS(DIFF).LT.OFF.AND.ABS(PNW(J)).LT.OFF) GO TO 80
         YINT=PN1(J)-SLOPE*RIT(J)
         PN1(J)=PNW(J)
         RIT(J)=RIRR(J)
         RIRR(J)=-YINT/SLOPE
         GO TO 50
80       RIT(J)=RIT(J)*100.
         RIRR(J)=RIRR(J)*100.
         WRITE (NPT,240) NAME(J),RIT(J),RIRR(J),KNT(J)
90       CONTINUE
```

```
      WRITE (NPT,260)
      READ (IPT,) IT
      IF(IT.EQ.0)STOP
      WRITE (NPT,270)
      READ (IPT,) RATI,RINCR
      RATI=RATI/100.
      RTD=1.+RATI/100.
      WRITE (NPT,250) (NAME(L),L=1,NA)
      DO 110 L=1,IT
         DO 100 J=1,NA
            PNW2(L,J)=0.0
            IYR=IY(J)
            DO 100 I=1,IYR
            K=IYEAR(I,J)
            RTE=RTD**K
100         PNW2(L,J)=PNW2(L,J)+RETURN(I,J)/RTE
         RATI=RATI*100.
         RATI2(L)=RATI
         WRITE (NPT,280) RATI,(PNW2(L,K),K=1,NA)
         RATI=(RATI+RINCR)/100.
110      RTD=1.+RATI
      WRITE (NPT,290)
      READ (IPT,160) IA
      IF (IA.EQ.'NO') STOP
      WRITE (NPT,300)
      READ (IPT,) XMAX,XMIN
      WRITE (NPT,310)
      READ (IPT,) YMAX,YMIN
      XRANGE=XMAX-XMIN
      YRANGE=YMAX-YMIN
      XSTEP=XRANGE/3
      YSTEP=YRANGE/3
      DO 120 I=1,7
         XAXIS(I)=(I-1)*XSTEP+XMIN
         YAXIS(I)=(I-1)*YSTEP+YMIN
120      CONTINUE
      CALL PRNPLOT (0,1,36,1,54,NPT,IMAGE)
      CALL PRNPLOT (1,XMIN,XMAX,YMIN,YMAX,0,IMAGE)
      CALL PRNPLOT (2,NX,NY,-4,1,0,IMAGE)
      DO 130 I=1,NA
         CALL PRNPLOT (3,ICHAR(I),RATI2,PNW2(1,I),IT,0,IMAGE)
130      CONTINUE
      WRITE (NPT,320)
      CALL PRNPLOT (4,30,IRAY,XAXIS,YAXIS,7,IMAGE)
      WRITE (NPT,330)
      STOP
C
140   FORMAT (//36H  NUMBER OF NET CASHFLOWS  (UP TO 4))
150   FORMAT (/19H  NAME OF CASHFLOW ,I2,21H (UP TO 8 CHARACTERS))
160   FORMAT (A8)
170   FORMAT (/35H  NUMBER OF NONZERO NET RETURNS IN ,A8)
180   FORMAT (/43H  LIST YEAR(S) AND AMOUNT(S) OF NET RETURNS)
190   FORMAT (/52H INPUT INITIAL INTEREST RATES FOR ITERATION(PERCENT))
200   FORMAT (//14H    CASHFLOW  ,A8)
210   FORMAT (/6X,4HYEAR,8X,5I10)
220   FORMAT (6X,14HNET RETURNS($),5F10.2)
230   FORMAT (A3)
240   FORMAT (/4X,9HCASHFLOW ,A8,17H HAS IRR BETWEEN ,F8.4,5H AND ,F8.4/
     14X,I3,25H ITERATIONS WERE REQUIRED//)
250   FORMAT (/1X,13HDISCOUNT RATE,20X,24H DISCOUNTED NET REVENUES/39X,1
     12H ALTERNATIVE/14X,4(6X,A8)//)
260   FORMAT (/50H   NUMBER OF INTEREST RATES FOR WHICH NPW IS TO BE,22H
     1CALCULATED(UP TO 100))
270   FORMAT (/49H   INITIAL INTEREST RATE AND STEP SIZE IN PERCENT)
280   FORMAT (3X,F8.4,3X,4F14.3)
290   FORMAT (//51H   WOULD YOU LIKE THE GRAPH OF THE CASHFLOW PROFILE)
300   FORMAT (//47H   MAXIMUM AND MINIMUM X - VALUES TO BE GRAPHED)
310   FORMAT (/47H   MAXIMUM AND MINIMUM Y - VALUES TO BE GRAPHED)
320   FORMAT (////)
330   FORMAT (//25X,22HINTEREST RATE(PERCENT)/)
340   FORMAT(3X,9HCASHFLOW ,A8,32H HAS NO IRR OR IRR WAS NOT FOUND/)
350   FORMAT(51H DO YOU WANT TO TRY WITH NEW INITIAL INTEREST RATES)
      END
```

```
RECOVER /SYSTEM: GET,IRR
READY.
X,IRR


  NUMBER OF NET CASHFLOWS  (UP TO 4)
? 2

  NAME OF CASHFLOW  1 (UP TO 8 CHARACTERS)
? INVEST 1

  NUMBER OF NONZERO NET RETURNS IN INVEST 1
? 4

  LIST YEAR(S) AND AMOUNT(S) OF NET RETURNS
? 0  -10,1  -15,2  5,3  20

 INPUT INITIAL INTEREST RATES FOR ITERATION(PERCENT)
? -5 -2


    CASHFLOW  INVEST 1

      YEAR                  0         1         2         3
      NET RETURNS($)     -10.00    -15.00      5.00     20.00

    CASHFLOW INVEST 1 HAS IRR BETWEEN    -.0091 AND    -.0000
      4 ITERATIONS WERE REQUIRED



  NAME OF CASHFLOW  2 (UP TO 8 CHARACTERS)
? INVEST 2

  NUMBER OF NONZERO NET RETURNS IN INVEST 2
? 4

  LIST YEAR(S) AND AMOUNT(S) OF NET RETURNS
? 0  10,1  15,2  -5,3  -20

 INPUT INITIAL INTEREST RATES FOR ITERATION(PERCENT)
? 0 10


    CASHFLOW  INVEST 2

      YEAR                  0         1         2         3
      NET RETURNS($)     10.00     15.00     -5.00    -20.00
   CASHFLOW INVEST 2 HAS NO IRR OR IRR WAS NOT FOUND

 DO YOU WANT TO TRY WITH NEW INITIAL INTEREST RATES
? YES

 INPUT INITIAL INTEREST RATES FOR ITERATION(PERCENT)
? -5 -0

    CASHFLOW INVEST 2 HAS IRR BETWEEN         0 AND         0
      2 ITERATIONS WERE REQUIRED
```

NUMBER OF INTEREST RATES FOR WHICH NPW IS TO BE CALCULATED(UP TO 100)
? 15

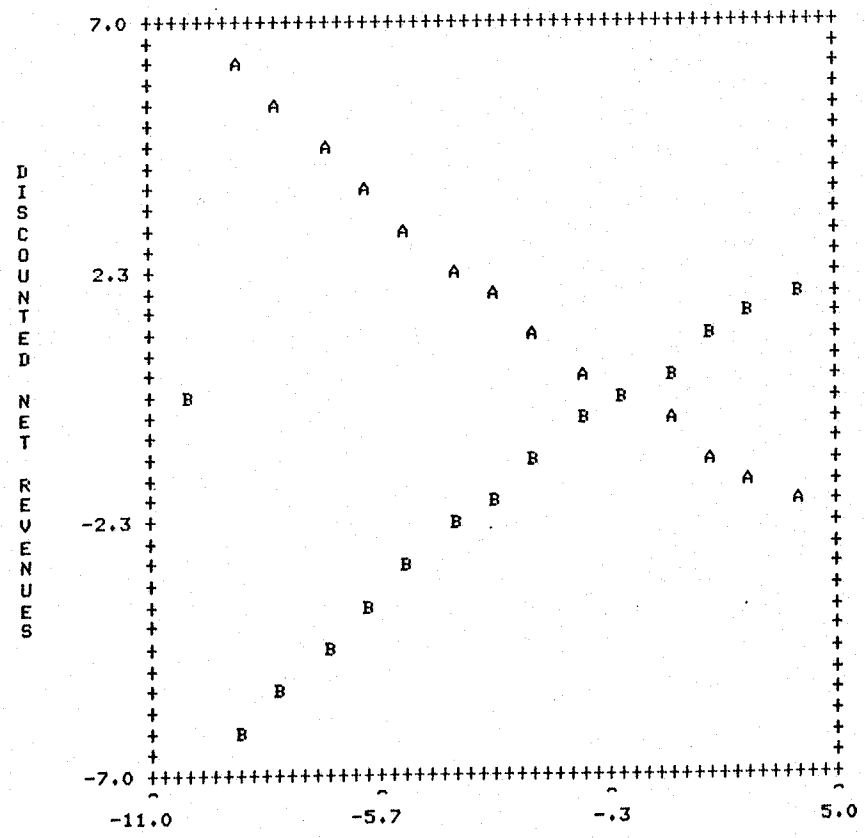
INITIAL INTEREST RATE AND STEP SIZE IN PERCENT
? -10 1

DISCOUNT RATE	DISCOUNTED NET REVENUES ALTERNATIVE	
	INVEST 1	INVEST 2
-10.0000	.055	-.055
-9.0000	6.095	-6.095
-8.0000	5.287	-5.287
-7.0000	4.517	-4.517
-6.0000	3.781	-3.781
-5.0000	3.078	-3.078
-4.0000	2.406	-2.406
-3.0000	1.764	-1.764
-2.0000	1.150	-1.150
-1.0000	.562	-.562
.0000	0	0
1.0000	-.538	.538
2.0000	-1.054	1.054
3.0000	-1.547	1.547
4.0000	-2.020	2.020

WOULD YOU LIKE THE GRAPH OF THE CASHFLOW PROFILE
? YES

MAXIMUM AND MINIMUM X - VALUES TO BE GRAPHED
? 5 -11

MAXIMUM AND MINIMUM Y - VALUES TO BE GRAPHED
? 7 -7



READY.
BYE

2219303 LOG OFF 16.23.14.
ABHA005 CP 0.875 SECS.