A Computer Program for Calculating
Internal Rate of Return and Present
Net Worth Profiles\textsuperscript{1/}

by

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March 1, 1978

STAFF PAPER SERIES NUMBER 1

\textsuperscript{1/} Research supported by the College of Forestry and the Agricultural Experiment Station, Institute of Agriculture, Forestry and Home Economics, University of Minnesota, St. Paul.

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Program IRR is a FORTRAN IV program that calculates the internal rate of return (IRR) for a specified series of net returns. The program also calculates present net worths (NPW) for a number of interest rates at specified intervals. It can be executed interactively over a computer terminal.

A Newton-Raphson iteration technique is used to find the internal rate of return. Given an initial interest rate $i_n$ and the NPW-function related to $i$, $f(i)$, the next interest rate $i_{n+1}$ in the iteration is found from:

$$i_{n+1} = i_n - \frac{f(i_n)}{f'(i_n)}$$

where $f'(i_n)$ is the derivative of the function at $i_n$ approximated in the program by the slope of a secant through the function $f(i)$ at two points or

$$f'(i_n) = \frac{f(i_{n+1}) - f(i_n)}{i_{n+1} - i_n}.$$  

This process is best illustrated by examining Figure 1 where the iteration was started with the two interest rates 2 and 10 percent. The slope of the secant through the two values of $f(2) = 60$ and $f(10) = -15$, representing the net present worths at the two assumed interest rates, is given by

$$f'(10) = \frac{f(10) - f(2)}{i_2 - i_1} = \frac{-15 - 60}{10 - 2} = \frac{-75}{8} = -9.38$$

The straight line with this slope will intersect the interest rate axis at the new trial interest rate:

$$i_3 = i_2 - \frac{f(i_2)}{f'(i_2)} = 10 - \frac{(-15)}{-9.38} = 10 - 1.6 = 8.4$$
Figure 1. Newton-Raphson Iteration

Figure 2. Possible Problems in Finding Roots of Function
and \( f(8.4) = -10 \). The next interest rate in the iteration would be 5.07. This process is continued until the functional value of present net worth is sufficiently close to zero.

Running the Program

Assuming that the compiled version of the program \( (\text{IRR}) \) has already been stored on a disk file, the execution of the program is quite simple. Shown below are control commands necessary for program execution. All user supplied inputs are underlined.

```
78/03/02, 15:42:11, TERMINAL - 206
ALL USERS-TYPE WRITEUP SYSNOTE (78/02/23),
USER NUMBER:
TERMINAL: 206, TTYD
PF STORAGE EXCEEDS LIMIT.
RECOVER /SYSTEM: GET,IRR
READY,
Y,IRR

NUMBER OF NET CASHFLOWS (UP TO 4)
? 1

NAME OF CASHFLOW 1 (UP TO 8 CHARACTERS)
? INVEST 1
```
Little has to be said about program inputs as they become self-explanatory at the time of program execution. The program when accessed via the previously described control commands will ask for relevant inputs. Examination of the attached sample run should give sufficient insight into the inputting procedure. All inputs except the names of the cash-flows are free-formated, i.e., can be entered without regard to size and number of digits after a decimal point or field on the input line or card. Individual numbers must be separated by one or more blanks or a comma.

Program output consists of

(1) the internal rate of return for each specified cashflow; sometimes the iteration procedure will not be able to find the internal rate of return because of the peculiar nature of a cashflow.

(2) a cashflow profile showing present net values for specified interest rates

(3) a graphical output of the cashflow profile. This output can only be requested if a cashflow profile was produced.

Cashflow profiles such as shown in Figure 2 can present a problem in finding an internal rate of return. Note that there are two solutions at which \( PNW = 0 \) and that \( PNW \) approaches the interest rate axis asymptotically on one side. If the two initial interest rates selected for the iteration are \( A \) and \( B \), the procedure will find one of the roots of the function. With \( B \) and \( C \) as starting values, the second root would have been found. With \( B \) and \( D \) as starting values, however, the iteration
process would solve for larger and larger new interest rates often leading to numbers that exceed the maximum word length of the computer. If an internal rate of return was not found by the program, new initial starting values for the interest rates can be used to find a solution (see sample run).

A number of program variables are initialized at values that one might want to change. \texttt{OFF} is a variable the value of which will determine when the iteration is to be terminated. Presently set at a value of 0.000 the iteration will stop as soon as the IRR is determined to the nearest 1/1000 of one percent as long as \texttt{NPW} is less than 0.1 dollar. For the latter, the parameter \texttt{OFF} is set equal to 0.1. The process will also be stopped if the slope (parameter \texttt{SLP}) becomes smaller than 50, i.e., present net worth does not change more than $0.05$ per 1 percent change in the interest rate, possibly indicating an asymptotic problem.

A program listing and example output are shown below.
PROGRAM IRR (INPUT, OUTPUT, TAPE1=INPUT, TAPE6=OUTPUT)
DIMENSION RETURN(100, 4), IYEAR(100, 4), PXW(4), PNI(4), NAME(4), NBR(4), RIT(4), RIRR(4), IY(4), KNT(4), RATE(4), PNW2(100, 4), NX(3), 2 NY(3), IMAGE(1200), IRAY(5), ICHAR(4), XAXIS(7), YAXIS(7), RATI2(3100)
DATA NX/4, 18, 1/, NY/4, 7, 1/
DATA IRAY/10H DISCOUNTED NE, OHT REVENUES/
DATA ICHAR/1HA, 1HB, 1HC, 1HD/
IPT=1
NPT=6
SLF=50.
OFF=0.1
OFF=0.0001
WRITE (NPT, 140)
READ (IPT, *) NA
DO 90 J=1, NA
  WRITE (NPT, 150) J
  READ (IPT, 160) NAME(J)
  WRITE (NPT, 170) NAME(J)
  READ (IPT, *) IY(J)
  WRITE (NPT, 180)
  READ (IPT, *) (IYEAR(I, J), RETURN(I, J), I=1, IY(J))
  WRITE (NPT, 190)
  READ (IPT, *) RATE(J), RIRR(J)
  I1=0
  I2=10
  WRITE (NPT, 200) NAME(J)
  I1=I1+1
  IF (I1 GT IY(J)) GO TO 30
  IF (IY(J)-5, LT, I1) I2=IY(J)
  WRITE (NPT, 210) (IYEAR(I, J), I=I1, I2)
  WRITE (NPT, 220) (RETURN(I, J), I=I1, I2)
  I1=I2
  I2=I2+10
  GO TO 20
20 PNW(J)=0.0
  RIT(J)=RATE(J)/100.
  DO 40 I=1, IY(J)
    K=IYEAR(I, J)
    RTE=(1.0+RIT(J))*K
    PNW(J)=PNW(J)+RETURN(I, J)/RTE
  40 PNI(J)=PNW(J)

C
C NEUTON-RAPHSON ITERATION
C
KNT(J)=0
RIRR(J)=RIRR(J)/100.
RIR=1.+RIRR(J)
PNW(J)=0.0
KNT(J)=KNT(J)+1
DO 60 I=1, IY(J)
  K=IYEAR(I, J)
  RTE=RIR**K
  PNW(J)=PNW(J)+RETURN(I, J)/RTE
  DIFF=RIT(J)-RIRR(J)
  IF (ABS(DIFF) LT OFF) GO TO 80
  SLOPE=(PNI(J)-PNW(J))/DIFF
  IF (ABS(SLOPE) GT SLP) GO TO 70
  WRITE (NPT, 340) NAME(J)
  WRITE (NPT, 350)
  READ (IPT, 230) REPLY
  IF (REPLY.EQ. 'NO') GO TO 90
  WRITE (NPT, 190)
  READ (IPT, *) RATE(J), RIRR(J)
  GO TO 30
70 IF (ABS(DIFF) LT OFF .AND. ABS(PNW(J)) LT OFF) GO TO 80
  YINT=PNI(J)-SLOPE*RIT(J)
  PNI(J)=PNW(J)
  RIT(J)=RIRR(J)
  RIRR(J)=-YINT/SLOPE
  GO TO 50
80 RIT(J)=RIT(J)*100.
  RIRR(J)=RIRR(J)*100.
  WRITE (NPT, 240) NAME(J), RIT(J), RIRR(J), KNT(J)
90 CONTINUE
WRITE (NPT,260)
READ (IPT,*) IT
IF (IT.EQ.0) STOP
WRITE (NPT,270)
READ (IPT,*) RATI,RINCR
RATI=RATI/100.
RINCR=1.+RATI/100.
WRITE (NPT,250) (NAME(L),L=1,NA)
DO 110 L=1,IT
   DO 100 J=1,NA
      PNW2(L,J)=0.0
      IYR=IY(J)
      DO 100 I=1,IYR
      K=IYEAR(I,J)
      RTE=RTE+K
      PNW2(L,J)=PNW2(L,J)+RETURN(I,J)/RTE
      RATI=RATI+100.
   100 RATI2(L)=RATI
   WRITE (NPT,280) RATI,(PNW2(L,K),K=1,NA)
   RATI=(RATI+RINCR)/100.
   110 RINCR=1.+RATI
WRITE (NPT,290)
READ (IPT,160) IA
IF (IA.EQ.'NO') STOP
WRITE (NPT,300)
READ (IPT,*) XMAX,XMIN
WRITE (NPT,310)
READ (IPT,*) YMAX,YMIN
XRANGE=XMAX-XMIN
YRANGE=YMAX-YMIN
XSTEP=XRANGE/3
YSTEP=YRANGE/3
DO 120 I=1,7
   XAXIS(I)=(I-1)*XSTEP+XMIN
   YAXIS(I)=(I-1)*YSTEP+YMIN
   120 CONTINUE
CALL PRNPLOT (0.1,36.1,54,NPT,IMAGE)
CALL PRNPLOT (1,XMIN,XMAX,YMIN,YMAX,0,IMAGE)
CALL PRNPLOT (2,NX,NY,-4,1,0,IMAGE)
DO 130 I=1,NA
   CALL PRNPLOT (3,ICHAR(I),RATI2,PNW2(1,I),IT,0,IMAGE)
   130 CONTINUE
WRITE (NPT,320)
CALL PRNPLOT (4,30,IRAY,XAXIS,YAXIS,7,IMAGE)
WRITE (NPT,330)
STOP
C
140 FORMAT (/36H NUMBER OF NET CASHFLOWS (UP TO 4))
150 FORMAT (/15H NAME OF CASHFLOW ,I2,21H (UP TO 8 CHARACTERS))
160 FORMAT (A8)
170 FORMAT (/35H NUMBER OF NONZERO NET RETURNS IN ,A8)
180 FORMAT (/43H LIST YEAR(S) AND AMOUNT(S) OF NET RETURNS)
190 FORMAT (/52H INPUT INITIAL INTEREST RATES FOR ITERATION(PERCENT))
200 FORMAT (/14H CASHFLOW ,A8)
210 FORMAT (/6X,4HYEAR,8X,5I10)
220 FORMAT (/6X,14HNET RETURNS(*),5F10.2)
230 FORMAT (A3)
240 FORMAT (/4X,9HCASHFLOW ,A8,17H HAS IRR BETWEEN ,F8.4,5H AND ,F8.4/
   14X,13.25H ITERATIONS WERE REQUIRED//)
250 FORMAT (/1X,13HDISCOUNT RATE,20X,24H DISCOUNTED NET REVENUES/39X,1
   12H ALTERNATIVE/14X,4(6X,A8)//)
260 FORMAT (/50H NUMBER OF INTEREST RATES FOR WHICH NPW IS TO BE,22H
   1CALCULATED(UP TO 100))
270 FORMAT (/49H INITIAL INTEREST RATE AND STEP SIZE IN PERCENT)
280 FORMAT (3X,F8.4,3X,4F14.3)
290 FORMAT (/51H WOULD YOU LIKE THE GRAPH OF THE CASHFLOW PROFILE)
300 FORMAT (/47H MAXIMUM AND MINIMUM X - VALUES TO BE GRAPHED)
310 FORMAT (/47H MAXIMUM AND MINIMUM Y - VALUES TO BE GRAPHED)
320 FORMAT (///)
330 FORMAT (/25X,22HINTEREST RATE(PERCENT)\//)
340 FORMAT(3X,9HCASHFLOW ,A8,32H HAS NO IRR OR IRR WAS NOT FOUND)!
350 FORMAT(51H DO YOU WANT TO TRY WITH NEW INITIAL INTEREST RATES)
END
RECOVER /SYSTEM: GET,IRR
READY,
X,IRR

NUMBER OF NET CASHFLOWS (UP TO 4)
? 2

NAME OF CASHFLOW 1 (UP TO 8 CHARACTERS)
? INVEST 1

NUMBER OF NONZERO NET RETURNS IN INVEST 1
? 4

LIST YEAR(S) AND AMOUNT(S) OF NET RETURNS
? 0 -10,1 -15,2 5,3 20

INPUT INITIAL INTEREST RATES FOR ITERATION(PERCENT)
? -5 -2

CASHFLOW INVEST 1

<table>
<thead>
<tr>
<th>YEAR</th>
<th>NET RETURNS($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>-10.00</td>
</tr>
<tr>
<td>1</td>
<td>-15.00</td>
</tr>
<tr>
<td>2</td>
<td>5.00</td>
</tr>
<tr>
<td>3</td>
<td>20.00</td>
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</tbody>
</table>

CASHFLOW INVEST 1 HAS IRR BETWEEN -.0091 AND -.0000
4 ITERATIONS WERE REQUIRED

NAME OF CASHFLOW 2 (UP TO 8 CHARACTERS)
? INVEST 2

NUMBER OF NONZERO NET RETURNS IN INVEST 2
? 4

LIST YEAR(S) AND AMOUNT(S) OF NET RETURNS
? 0 10,1 15,2 -5,3 -20

INPUT INITIAL INTEREST RATES FOR ITERATION(PERCENT)
? 0 10

CASHFLOW INVEST 2

<table>
<thead>
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<th>YEAR</th>
<th>NET RETURNS($)</th>
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<tbody>
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</tr>
<tr>
<td>2</td>
<td>-5.00</td>
</tr>
<tr>
<td>3</td>
<td>-20.00</td>
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</tbody>
</table>

CASHFLOW INVEST 2 HAS NO IRR OR IRR WAS NOT FOUND

DO YOU WANT TO TRY WITH NEW INITIAL INTEREST RATES
? YES

INPUT INITIAL INTEREST RATES FOR ITERATION(PERCENT)
? -5 -0

CASHFLOW INVEST 2 HAS IRR BETWEEN 0 AND 0
2 ITERATIONS WERE REQUIRED
NUMBER OF INTEREST RATES FOR WHICH NFW IS TO BE CALCULATED (UP TO 100)

? IS

INITIAL INTEREST RATE AND STEP SIZE IN PERCENT

? -10 1

<table>
<thead>
<tr>
<th>DISCOUNT RATE</th>
<th>INVEST 1</th>
<th>DISCOUNTED NET REVENUES</th>
</tr>
</thead>
<tbody>
<tr>
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<td></td>
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<tr>
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</tr>
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<tr>
<td>4.0000</td>
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</table>

WOULD YOU LIKE THE GRAPH OF THE CASHFLOW PROFILE

? YES

MAXIMUM AND MINIMUM X - VALUES TO BE GRAPHED

? 5 -11

MAXIMUM AND MINIMUM Y - VALUES TO BE GRAPHED

? 7 -7

INTEREST RATE (PERCENT)