# INTERMEDIATE INPUTS AND TECHNICAL CHANGE IN THE

U.S. LUMBER AND WOOD PRODUCTS INDUSTRY 1

by

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#### Abstract

This paper examines the estimation of technical change in the U.S. lumber and wood products industry. Previous studies along these lines have assumed an industry production function in which output is measured as value-added, and therefore intermediate inputs are not treated symetrically with capital and labor inputs. The exclusion of intermediate inputs for the production process in industry level productivity studies places unnecessary restrictions on the production process, producer behavior and the nature of technical change. It is shown that the estimated rate of technical change based on a value-added model of production is necessarily greater than the rate derived from a model that treats all inputs symetrically. The practical significance of this upward bias is examined by calculating indexes of technical change for the lumber and wood products industry based on value-added and gross output production functions. It is concluded that the gross output model is more appropriate for analyzing productivity and technical change in the lumber and wood products industry.

# Intermediate Inputs and Technical Change in the Lumber and Wood Products Industry

#### Introduction

Technical change in primary resource using industries can be a major factor affecting the utilization and value of those resources. Technical change enables less expensive or relatively abundant resources to be substituted for expensive or scarce resources. A current example in the forest products industry is the substitution of structural particleboard — made primarily with low value aspen — for softwood plywood. Since technical change redefines and expands resources, its analysis and estimation may be of interest to policymakers and resource managers.

The analysis of technical change in the forest products industry is a relatively recent area of empirical research. Manning and Thornburn (1971) estimate the rate of Hicks-neutral technical change in the Canadian pulp and paper industry using a model developed by Solow (1957). The same model was applied to the U.S. lumber and wood products industry (SIC 24) by Robinson (1975). Using a cost function approach developed by Binswanger (1974), Stier (1980b) examined factor substitution and technical change in the U.S. lumber industry (SIC 242). Three factors of production were included in this model: capital, labor and sawlogs. A two factor cost function model was used to examine the nature of technical change in ten U.S. forest products industries (Stier, 1980a). Greber and White (1982) analyzed non-neutral technical

change in lumber and wood products using a model developed by Sato (1970).

The Manning and Thornburn, Robinson, and Greber and White studies were based on models originally developed to analyze technical change in the aggregate U.S. economy. With the exception of Stier (1980b), all of the forest products studies mentioned in the preceeding paragraph postulate a two factor industry production function in which output (measured as value-added) is a function only of capital and labor, and thus intermediate inputs are not treated symetrically with capital and labor inputs. The exclusion of intermediate inputs from the production process in industry level productivity studies has come under increasing criticism in recent years. It has been shown (Berndt and Christensen, 1973; Gollop, 1979) that the value-added model of production places unnecessary restrictions on the production process, producer behavior and the nature of technical change when applied to an individual industry. Furthermore, Gollop has shown that the rate of industry technical change obtained from a value-added model is greater than the rate derived from a model that treats all inputs symetrically. Hence, the rates of technical change estimated in previous studies by may be biased upward if the restrictions implied by the value-added model are inappropriate.

This paper will examine the analysis of technical change at the industry level -- specifically, the U.S. lumber and wood products industry -- with special attention focused on the role of intermediate inputs. The restrictions introduced by the value-added model will be

discussed, and the upward bias in the index of technical change will be demonstrated. The practical significance of this bias will be shown by reformulating the Robinson study using a gross output model which includes an explicit measure of intermediate input.

## Restrictions of the Value-Added Model

Intermediate inputs are defined as purchases of goods and services by one firm or industry from another. Examples include raw materials (including wood) and fuel. When the production accounts of all industries are aggregated at the national level, inter-industry flows of intermediate inputs cancel out: the intermediate input of one industry is the output of some other industry. Therefore, the final output of an entire economy is produced only by the combination of primary inputs (capital and labor), given some technology (National Research Council, 1979).

At the level of an individual firm or industry, however, intermediate inputs are relevant and must be included in the production function. In some industries -- including lumber and wood products -- intermediate inputs represent a larger proportion of the value of output than capital and labor inputs. The data presented in Table 1 indicates that the value share of intermediate inputs has averaged about .55 in the U.S. lumber and wood products industry.

The most general way to include intermediate inputs is to specify an unrestricted industry production function

(1) 
$$Q = f(K, L, X, t),$$

where Q is the gross output of the industry, K, L and X are capital, labor and intermediate inputs respectively, and t represents time. Each of these variables except t is an aggregation of specific outputs or inputs. This model of production is referred to as a gross output model.

Alternatively, a value-added industry production function can be written as

(2) 
$$V = g(K, L, t),$$

where V is the value-added by the industry, defined as gross output less intermediate inputs. Implicit in the value-added model is the assumption that the function g is separable from X, i.e., we can write

$$Q = f[g(K,L,t),X]$$

This is termed value-added separability. Note that both the gross output model and the value-added model include a measure of intermediate inputs. The difference lies in the way in which intermediate inputs are treated.

Value-added separability implies restrictions on the production process, producer behavior and technical change. With regard to the production process, separability implies that the marginal rate of substitution between capital and labor be independent of the level of intermediate input (Denny and Fuss, 1977). Alternatively, Berndt and Christensen (1973) have shown that value-added separability requires that the Allen partial elasticities of substitution between a factor in the sub-function g and intermediate input be equal for all factors in g (i.e.,  $\mathcal{C}_{kx} = \mathcal{C}_{1-}$ ).

This restriction, then, would effectively rule out the possibility of economies of scale with regard to the intermediate input, an assumption which may be inappropriate. Recent energy innovations in the lumber and wood products industries have most likely resulted in significant economies of scale with respect to fossil fuels — the primary energy intermediate input over time. In these industries there has been an increasing substitution of wood residues for fossil fuels as an energy source. A firm's stock of internally generated wood residues is directly related to production level. If this source of energy is insufficient to cover needs, additional energy, in the form of wood residues and fossil fuels, must be purchased — a charge to intermediate input.

The implied restrictions on the production process in the value—added model also leave much to be desired in terms of intuitive appeal, as emphasized by Domar: "a production function is supposed to explain a production process, such as the making of potato chips from potatoes (and other ingredients), labor, and capital. It must take some ingenuity to make potato chips without potatoes," (1967, p. 471).

The value-added model restricts producers from equating the price of intermediate inputs to their value of marginal product (VMP), as required for profit maximization. It is implicitly assumed that the ratio of intermediate inputs to output remains constant, regardless of the price of intermediate inputs. Gollop (1979) provides a compelling intuitive illustration of the restrictions on producer behavior implied by value-added separability:

A supermarket manager can choose to have his own employee display frozen ice cream products in the frozen foods cabinet or he may contract with the raw materials supplier to have its delivery person display the product. The former is a direct labor cost to the supermarket; the latter is an expense related to intermediate input. Presumably, the store manager makes this choice such that the ratio of marginal products equals the corresponding ratio of factor prices. Imposing value-added separability unnecessarily restricts the production function describing the supermarket's operation, the characterization of the manager's rational behavior, and the necessary conditions for producer equilibrium, (pp. 321-2).

The characterization of technical change is also limited by the value-added model. Hulten (1978) has pointed out that the value-added approach assigns all measured technical progress to capital and labor inputs. Technical change is allowed to affect output only through the function g, and thus intermediate inputs are ruled out as a source of productivity growth. This is a particularly restrictive assumption for some of the lumber and wood products industries where technical change is often embodied in new products which enter into the production process as production inputs, i.e., intermediate inputs. For example, technological changes in forest management — intensive silvicultural practices, genetic improvements, etc. — or timber harvesting methods would be manifested in the timber resource.

In a wood resource using industry, the assumption that technical change has not affected the efficiency of intermediate input use is likely to be inappropriate. Several studies have found sawlogs to be the only natural resource commodity exhibiting increasing economic scarcity over the period from 1870 to 1973 (Ruttan and Callaham, 1962; Barnett and Morse, 1963; Manthy, 1977).

In light of this information, one might expect based on induced innovation theory (Binswanger and Ruttan, 1978) that at least some

technical change in lumber and wood products has had the effect of conserving the relatively expensive stumpage input. In fact, Stier (1980) found a slight log-saving bias in technical change in the lumber industry (SIC 242). Stier's estimate of this bias is conservative if the declining quality of sawlogs is understated by a log scale. Spelter (1980) has found that saw log overruns have been increasing overtime as average log diameters decline. This suggests that the Stier estimates are in fact biased downward.

Many recent technological advances in forest products have had the effect of increasing lumber and plywood recovery per unit of cubic log input (Haygreen and Bowyer, 1982, p. 442), so it is likely that the wood-saving bias will increase in the future. Therefore, restricting technical change from affecting the efficiency of intermediate input use in the forest products industry is highly questionable.

#### Upward Bias of the Value-Added Model

It has been shown that value-added separability implies a number of restrictions which may not be appropriate, particularly in the lumber and wood products industry. This section demonstrates that the rate of technical change based on a value-added model of production is necessarily larger than the rate obtained from a gross output model, given consistent data.

The following relationship, based on a value-added model of production, was used by Robinson for measuring neutral technical change:

(4) 
$$(A/A)^* = (V/V) - v_k (K/K) - v_1 (L/L),$$

where A measures cumulative shifts in the production function over time, V is net output or quantity of value-added, K is capital input, L is labor input,  $v_k^*$  is capital's share of income,  $v_1^*$  is labor's share of income, and the dots indicate time derivatives. Note that  $v_k^* = (p_k \text{ K/q*V})$ , where  $p_k$  is the price of capital inputs and  $q^*$  is the price of net output, and  $v_1^* = (p_1 \text{ L/q*V})$ , where  $p_1$  is the price of labor inputs. See Robinson (1975, pp. 149-50) for the derivation of this relationship.

An analogous relationship for measuring neutral technical change based on a gross output model is

(5) 
$$(\dot{A}/A) = (\dot{Q}/Q) - v_k(\dot{K}/K) - v_1(\dot{L}/L) - v_y(\dot{X}/X),$$

where Q is gross output of the industry, X is intermediate input, and  $\mathbf{v}_{\mathbf{x}}$  is intermediate input's share of income. Note that  $\mathbf{v}_{\mathbf{x}} = (\mathbf{p}_{\mathbf{x}} \ \mathbf{x}/\mathbf{qQ})$ , where  $\mathbf{p}_{\mathbf{x}}$  is the price of intermediate input and q is the price of output, and  $\mathbf{v}_{\mathbf{k}}$ ,  $\mathbf{v}_{\mathbf{l}}$  are defined analogously to  $\mathbf{v}_{\mathbf{x}}$ . See Appendix A for the derivation of this expression.

Proof that the industry rate of neutral technical change derived from the value-added production function is greater than the corresponding rate derived from the gross output model follows from a

comparison of (A/A) and (A/A)\*. Following Gollop's (1979) reasoning for a more general case, the definition of the value of an industry's net output or value-added is

$$q*V = qQ - p_{v}X.$$

Totally differentiating (6) with respect to time, dividing both sides by q\*V and simplifying yields

(7) 
$$(\dot{V}/V) = (qQ/q*V) (\dot{Q}/Q) - (p_x X/q*V) (\dot{X}/X).$$

Substituting (7) into (4), we get:

(8) 
$$(\dot{A}/A) * = (qQ/q*V) (\dot{Q}/Q) - (p_X/q*V) (\dot{X}/X)$$

$$- (p_k K/q*V) (\dot{K}/K) - (p_1 L/q*V) (\dot{L}/L).$$

Note that if we multiply (8) by (q\*V/qQ), we obtain (5). Therefore,

(9) 
$$(\dot{A}/A) * = (qQ/q*V) (\dot{A}/A)$$

$$= (qQ/(qQ - p_{_{Y}}X)) (\dot{A}/A).$$

Since the ratio of value of gross output to the value of net output must be greater than one, i.e.,

$$(qQ/(qQ - p_xX)) > 1,$$

it follows that  $(\dot{A}/A)* > (\dot{A}/A)$ . Therefore, the rate of technical change estimated from the value-added model is greater than the corresponding rate estimated from the gross output model. Notice that  $(\dot{A}/A)$  is equal to  $(\dot{A}/A)*$  less an amount proportional to the industry's share of intermediate input purchases in its total factor payments:

(10) 
$$(\dot{A}/A) = (1 - (p_X X/qQ)) (\dot{A}/A) *$$

$$= (\dot{A}/A) * - v_X (\dot{A}/A) * .$$

Thus, the larger the income share of intermediate inputs in an industry, the larger the gap between the two measures of technical change will be.

When the prices of intermediate inputs such as stumpage and energy are

rising, an index of technical change based on the value-added model will rise relative to an index based on the gross output model.

## An Empirical Example

What is the practical significance of the upward bias in an index of technical change introduced by the value-added model? In order to address this question, Robinson's index was recalculated and compared to an index based on the gross output model. These indexes were obtained by first calculating  $(\mathring{A}/A)$  and  $(\mathring{A}/A)^*$ , and then setting A(1) equal to one and evaluating the expression

$$A(t+1) = A(t) [1 + (A/A)],$$
  $t = 1, 2, ..., T$ 

for the index based on the gross output model, and

$$A(t+1) = A(t) \left[1 + (A/A)^*\right], \qquad t = 1, 2, ..., T$$
 for the index based on the value-added model.

A meaningful comparison requires that the data employed in each index are consistent. Robinson's data are flawed in several respects (see, pp. 151-52), so data on capital and labor inputs reported in Greber and White (1982) were used. Unpublished data collected by the Bureau of Economic Analysis (1974) on intermediate input, value added and gross output (SIC 24) were also used. This is the same intermediate input data used by Gollop and Jorgenson (1980) in their mammoth study of U.S. productivity growth by industry. The complete data set is presented in Table 1.

The calculated indexes are shown in Table 2 and Figure 1. It is clear that the large income share of intermediate inputs in the lumber and wood products industry creates a significant gap between the two measures of productivity. This gap widens when the importance of intermediate inputs in production is rising.

Note in Figure 1 that the value-added index A(t)\* exhibits a sawtooth pattern -- peaks and valleys are more pronounced than in the gross output index A(t). A similar pattern was observed by Solow (1957, p. 316). The value-added index thus exagerates both increases and decreases in total factor productivity due to its treatment of intermediate inputs.

### Concluding Remarks

Much of the early empirical analysis of productivity and technical change was done at the economy-wide level, and thus it was reasonable to work with a value-added model of production. The application of this same approach to individual industries or sectors has been common (e.g., Lydall, 1968; Doutriaux and Zind, 1976; Lianos, 1976; Batavia, 1979), often due to a lack of data on intermediate inputs. But the restrictive assumptions of the value-added model have come under increasing attack in recent years.

These assumptions are particularly suspect in a wood resource using industry. As noted recently by the National Research Council (1979),

... when the price of some materials rise sharply, as occurred recently for wood and more dramatically for coal and oil, industries that use these materials substitute cheaper materials if possible or they cut down on their use by reducing waste (which often involves substituting capital or labor). The probable result of these economizing actions

is that the ratio of the value of intermediate inputs to gross output changes (p. 141).

In wood using industries, rising real stumpage prices have induced producers to reduce intermediate input relative to output by utilizing as materials or fuel residuals that were formerly discarded, and by adopting new production technologies that reduce wood requirements (Myers and Nakamura, 1979). Thus, the assumptions of the value-added model are clearly violated: the ratio of intermediate input to output is not constant, and technical change does affect the efficiency of intermediate input utilization. It is therefore concluded that the gross output approach is more appropriate for analyzing productivity and technical change in the lumber and wood products industry.

Table 1. Data series for the U.S. Lumber and Wood Products Industry, (SIC 24), 1951-1973 (all dollar amounts in millions of 1958 dollars).

Year	<u>к</u> а/	ќ/к	<u>L</u> b/	Ľ/L	<u>x</u> c/	*/x
1951	5,297.25		842		3,589	
1952	5,468.99	.0324	783	0701	3,620	.0086
1953	5,555.67	.0158	751	0409	3,773	.0423
1954	5,657.06	.0182	683	0905	3,719	0143
1955	5,817.23	.0283	719	.0527	4,228	.1369
1956	6,072.51	.0439	706	0181	4,227	0002
1957	6,060.24	0020	631	1062	3,983	<b></b> 0577
1958	6,198.63	.0228	583	0761	4,360	.0947
1959	6,231.67	.0053	623	.0686	4,784	.0972
1960	6,401.80	.0273	601	0353	4,832	.0100
1961	6,451.67	.0078	557	0732	4,891	.0121
1962	6,531.77	.0124	566	.0162	5,321	.0879
1963	6,652.68	.0185	567	.0018	5,558	.0445
1964	6,728.57	.0114	582	.0265	5,361	0354
1965	6,817.62	.0132	595	.0223	5,429	.0127
1966	7,014.41	.0289	603	.0134	5,313	0217
1967	7,071.61	.0082	581	0365	5,500	.0352
1968	7,110.31	.0055	587	.0103	5,463	0067
1969	7,292.76	.0257	588	.0017	5,492	.0055
1970	7,435.53	.0196	559	0493	5,633	.0257
1971	7,497.84	.0084	573	.0250	5,560	0130
1972	7,610.20	.0150	603	.0523	6,673	.2002
1973	7,588.11	0029	623	.0332	6,434	0358
						•

 $<sup>\</sup>frac{a}{}$  Capital input. Source: Greber and White, 1982.

b/ Labor input, thousands of full-time equivalent employees.
Source: National Income and Products Accounts of the United States,
1929,1974 (reported in Greber and White, 1982).

c/ Cost of materials. Source: Bureau of Economic Analysis (1974).

Table 1. (continued)

Year	Qd/	Q/Q	<u>ve</u> /	v/v	
			*		·
1951	6,948		3,460		
1952	6,962	.0020	3,438	0064	
1953	7,040	.0112	3,333	0305	
1954	6,711	0467	3,031	0906	
1955	7,659	.1412	3,478	.1475	
1956	7,717	.0076	3,544	.0190	
1957	7,178	0698	3,231	0883	
1958	7,525	.0483	3,165	0204	
1959	8,215	.0917	3,415	.0790	
1960	8,043	0209	3,157	0755	
1961	8,001	0052	3,031	0399	
1962	8,432	.0539	2,992	0129	
1963	9,520	.1290	3,952	.3209	
1964	10,010	.0515	4,746	.2009	
1965	10,190	.0180	4,865	.0251	
1966	10,223	.0032	5,048	.0376	
1967	10,779	.0544	5,450	.0796	
1968	11,046	.0248	5,800	.0642	
1969	10,758	0261	5,445	0612	
1970	11,030	.0253	5,567	.0224	
1971	11,017	0012	5,593	.0047	
1972	13,132	.1920	6,682	.1947	
1973	12,702	0327	6,483	0298	

 $<sup>\</sup>frac{d}{}$  Value of production. Source: Bureau of Economic Analysis (1974).

e' Value-added. Source: Bureau of Economic Analysis (1974).

Table 1. (continued).

Year	v <sub>k</sub>	<b>v</b> <sub>1</sub>	v <sub>x</sub>	v <sub>k</sub> *	<b>v</b> <sub>1</sub> *	
1951	.1766	.3227	•5007	.3536	.6464	·
1952	.1548	.3312	.5140	.3184	.6816	
1953	.1548	.3255	.5197	.3222	.6778	
1954	.1336	.3292	.5372	.2887	.7113	
1955	.1434	.3123	•5443	.3146	.6854	
1956	.1465	.3132	•5403	.3187	.6813	
1957	.1261	.3216	•5523	.2816	.7184	
1958	.1171	.3107	•5722	.2613	.7387	
1959	.1189	.3046	.5765	.2807	.7193	
1960	.1017	•3155	.5828	.2437	.7563	
1961	.1052	.3153	•5795	.2502	.7498	
1962	.1063	.3153	.5784	. 2494	.7506	
1963	.1266	.3164	.5570	.3287	.6713	
1964	.1425	.3018	.5557	.3091	.6909	
1965	.1292	.3070	.5638	.2840	.7160	
1966	.1288	.3139	•5573	.2910	.7090	
1967	.1373	.3069	•5558	.3093	.6907	
1968	.1631	.2963	• 5406	.3551	.6449	
1969	.1643	.2912	• 5445	.3606	.6394	2.*
1970	.1307	.3211	.5482	.2892	.7108	
1971	.1333	.3124	.5543	.2990	.7010	
1972	.1825	.2621	•5554	.4106	.5894	
1973	.2014	.2471	•5515	•4490	.5510	

Definitions provided within the text.

Table 2. Residuals and indexes of technical change for the gross output and value-added models (definitions provided within the text).

Year	Å/A	(Å/A)*	A(t)	A(t)*	
1951	0150	0011	1.0000	1.0000	
1952	.0158	.0311	1.0158	1.0311	
1953	0117	0079 0315	1.0159	1.0311	
1954	.0462	.1025	1.0040	.9908	
1955	.0069	.0173	1.0504	1.0924	
1956	0035	0117	1.0576	1.1113	
1957	.0151	.0299	1.0539	1.0983	
1958	.0141	.0282	1.0698	1.1311	
1959	0184	0555	1.0849	1.1630	
1960 1961	.0100	.0130	1.0649	1.0985	
1962	0034	0282	1.0755 1.0718	1.1128	
1963	.1013	.3136	1.1804	1.4205	
1964	.0615	.1791	1.2530	1.6749	
1965	.0023	.0054	1.2559	1.6839	
1966	.0074	.0197	1.2652	1.7171	
1967	.0449	.1023	1.3220	1.8928	
1968	.0164	.0556	1.3437	1.9980	
1969	0337 .0245	0716 .0518	1.2984	1.8549	
1970	0029	0153	1.3302	1.9510	
1971	.0644	.1577	1.3263	1.9211	
1972	0206	0494	1.4117	2.2241	
1973			1.3826	2.1142	

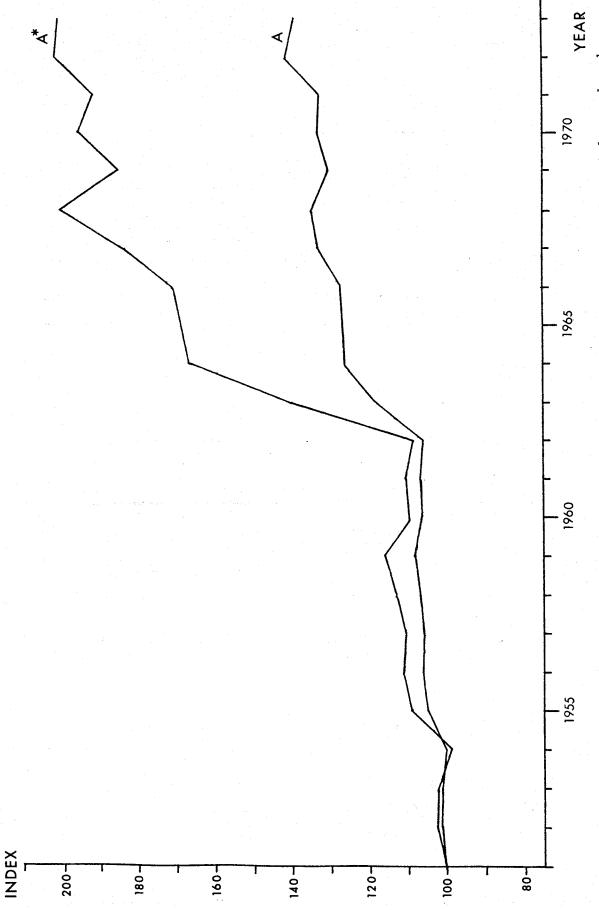


Figure 1. Indexes of technical change for the U.S. lumber and wood products industry based on value added (A\*) and gross output (A) models of production, 1951-1973, (1951=100).

## Appendix A

Derivation of the relationship used to estimate the rate of neutral technical change based on a gross output production function is presented in this appendix. This approach is an extension of Solow's (1957) "residual" method, in which changes in productivity are attributed to changes in output which are not explained by changes in inputs. The extension is the explicit inclusion of intermediate inputs, in addition to capital and labor.

The unrestricted industry production function can be written as Q = f(K, L, X, t),

where Q is gross output of the industry, K, L and X are capital, labor and intermediate inputs respectively and t is time. If technical change is assumed to be neutral, the production function takes the form

(2) 
$$Q = A(t)f(K, L, X),$$

where A(t) measures the cumulative shifts in the production function due to technical change over time. Totally differentiating (2) with respect to time, dividing both sides by Q and simplifying yields

- (3)  $(\dot{Q}/Q) = (\dot{A}/A) + A(\partial f/\partial K) (\dot{K}/Q) + A(\partial f/\partial L) (\dot{L}/Q) + A(\partial f/\partial X) (\dot{X}/Q)$ , where the dots indicate time derivatives. Since  $(\partial Q/\partial K) = A(\partial f/\partial K)$ ,  $(\partial Q/\partial L) = A(\partial f/\partial L)$  and  $(\partial Q/\partial X) = A(\partial f/\partial X)$ , equation (3) may be written as
- (4)  $(\dot{Q}/Q) = (\dot{A}/A) + (\partial Q/\partial K) (K/Q) (\dot{K}/K) + (\partial Q/\partial L) (L/Q) (\dot{L}/L) + (\partial Q/\partial X) (X/Q) (\dot{X}/X).$

If factors of production are paid their marginal products and the production function is assumed to be homogeneous of degree one in inputs, then

(5) 
$$G_k = (20/3K) (K/Q) = v_k$$

(6) 
$$C_1 = (\partial Q/\partial L) (L/Q) = v_1$$

(7) 
$$G_{x} = (20/3x) (x/0) = v_{x}$$

(8) 
$$v_k + v_1 + v_x = 1$$

where  $\mathcal{C}_k$ ,  $\mathcal{C}_1$  and  $\mathcal{C}_x$  are the elasticities of output with respect to capital, labor and intermediate input respectively, and  $\mathbf{v}_k$ ,  $\mathbf{v}_1$  and  $\mathbf{v}_x$  are the respective value shares of inputs. Substituting (5), (6) and (7) into (4) and rearranging terms we obtain

(9) 
$$(\dot{A}/A) = (\dot{Q}/Q) - v_k (\dot{K}/K) - v_1 (\dot{L}/L) - v_x (\dot{X}/X)$$

This expression can be calculated using discrete approximations of the time derivatives. The index of technical change is then obtained by setting A(1) equal to one and evaluating the expression

(10) 
$$A(t+1) = A(t) [1 + (\dot{A}(t)/A(t))], t = 1, 2, ..., T$$

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